Computing DERL for the sBG Model Using Excel

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1 Introduction

Fader and Hardie (2007a) introduce the notion of *discounted expected residual lifetime* (DERL) as a key element in the calculation of customer lifetime value in a contractual setting. When contract duration is characterized by the shifted-beta-geometric (sBG) distribution, the expression for DERL (when calculated just before the point in time at which the contract renewal decision is made) for a customer who has made $n-1$ contract renewals is (Fader and Hardie 2007a, equation 6)

$$DERL(d \mid \alpha, \beta, \text{active in } n) = \left(\frac{\beta+n-1}{\alpha+\beta+n-1}\right) {}_2F_1\left(1, \beta+n; \alpha+\beta+n; \tfrac{1}{1+d}\right). \quad (1)$$

This is easy to compute in any modeling environment that contains a routine for evaluating the Gaussian hypergeometric function, ${}_2F_1(\cdot)$. Even when such a modeling environment is not available, it is not too difficult to compute DERL. This note shows how to evaluate an expression equivalent to (1) using a simple Microsoft Excel worksheet.

Consider the task of estimating the residual value of the customer base for a hypothetical firm where the history of the customer base is documented in Table 1. This pattern of customer retention is generated by the sBG model with $\alpha = 3.8$ and $\beta = 15.2$. (In practice, we wouldn't know the model parameters and would have to estimate them given data of the form presented in Table 1 — see Fader and Hardie (2007b).) We first describe

how to compute the discounted expected residual lifetime of an individual customer using a simple Excel worksheet (Section 2). We then show how to extend this to the task of computing the residual value of the entire customer base (Section 3).

2002	2003	2004	2005	2006
10,000	8,000	6,480	5,307	4,391
	10,000	8,000	6,480	5,307
		10,000	8,000	6,480
			10,000	8,000
				10,000
10,000	18,000	24,480	29,787	34,178

Table 1: Number of active customers each year by year-of-acquisition

2 Computing DERL

One approach to the derivation of (1) starts in the following manner:

$$\begin{aligned} DERL(d \,|\, \alpha, \beta, \text{active in } n) &= \sum_{t=n}^{\infty} \frac{S(t \,|\, \alpha, \beta, t > n-1)}{(1+d)^{t-n}} \\ &= \sum_{t=n}^{\infty} \frac{S(t \,|\, \alpha, \beta)}{S(n-1 \,|\, \alpha, \beta)} \Big(\frac{1}{1+d}\Big)^{t-n} . \end{aligned} \tag{2}$$

If we can easily compute the survivor function associated with the sBG model, we can evaluate (2), terminating the series at a point where additional terms are effectively zero. Noting at the (aggregate) retention rate for period t under the sBG model is

$$r_t = \frac{\beta + t - 1}{\alpha + \beta + t - 1} , \tag{3}$$

we can compute the survivor function using the expression

$$S(t) = \prod_{i=1}^{t} r_i . \tag{4}$$

Let us consider the task of computing DERL for one of the 6,480 customers who has made two contract renewals (i.e., has been active for $n = 3$ periods).

- Assuming the values of α and β are located in cells `B1:B2`, respectively, we can compute $S(t)$ for the first, say, 200 periods in the following manner — see Figure 1 or the `Calculating DERL` worksheet in the Excel spreadsheet `sBG_DERL_using_Excel.xls`):

  - We enter the values of $t = 0, 1, 2, \ldots, 200$ in cells `A5:A205`.
  - Noting that $S(0) = 1$ (cell `B5`), we compute $S(1)$ by entering `=(B$2+A6-1)/(B$1+B$2+A6-1)*B5}` in cell `B6`.
  - We copy `B6` to `B7:B205`.

- The next step is to compute the conditional survivor function $S(t \mid t > 2)$, which is simply $S(t)/S(2)$ for $t > 2$.
  - We compute $S(3 \mid t > 2)$ by entering `=B8/$B$7` in cell `D8`.
  - We copy `D8` to `D9:D205`.

- The final component of (2) is the discount factor. Given a 10% discount rate, we enter `=1/1.1^($A8-3)` in cell `E8` and copy it to `E9:E205`.

- Entering `=SUMPRODUCT(D6:D205,E6:E205)` in cell `D1` gives us the value of DERL for a customer who has made two contract renewals (evaluated just before the point in time at which the third contract renewal decision is to be made).

	A	B	C	D	E	F
1	alpha	3.8	**DERL**	3.59		
2	beta	15.2				
3				2 renewals (n=3)		
4	t	S(t)		S(t|t>n-1)	disc.	
5	0	1.0000				
6	1	0.8000				
7	2	0.6480				
8	3	0.5307		0.8190	1.0000	
9	4	0.4391		0.6776	0.9091	
10				0.5656	0	
11				0.4761	0.7513	
12	7	0.2616		0.4037	0.6830	
13	8	0.2234		0.3447	0.6209	
14	9	0.1919		0.2962	0.5645	
15	10	0.1659		0.2560	0.5132	
201	196	6.14E-05		9.48E-05	1.03E-08	
202	197	6.03E-05		9.31E-05	9.33E-09	
203	198	5.93E-05		9.15E-05	8.48E-09	
204	199	5.82E-05		8.99E-05	7.71E-09	
205	200	5.72E-05		8.83E-05	7.01E-09	

Figure 1: Screenshot of Excel worksheet for computing DERL

Terminating the series at 200 instead of running it out to infinity underestimate the true value of DERL by 5.21E−12; this error should be acceptable in most settings!

3 Valuing a Customer Base

If we wish to value the entire customer base as reported in Table 1, we need to compute DERL for $n = 1, \ldots, 5$. This is done in the `DERL for each cohort` worksheet, following the logic documented in the preceding section.

With reference to the `Residual Value of Customer Base` worksheet, we compute a customer's expected residual lifetime value by multiplying his DERL by the expected net cashflow per period ($\bar{v}$):

$$E[RLV(d \,|\, \text{active in } n)] = \bar{v} DERL(d \,|\, \text{active in } n)\,.$$

(See cells `G4:G8`.) Assuming $\bar{v} = \$1$, the expected residual value of the entire customer base is $4{,}391 \times \$3.84 + 5{,}307 \times \$3.72 + 6{,}480 \times \$3.59 + 8{,}000 \times \$3.45 + 10{,}000 \times \$3.31 = \$120{,}543$, which is computed in cell `G11` using the formula `=SUMPRODUCT(E4:E8,G4:G8)`.

References

Fader, Peter S. and Bruce G. S. Hardie (2007a), "Customer-Base Valuation in a Contractual Setting: The Perils of Ignoring Heterogeneity." <http://brucehardie.com/papers/022/>

Fader, Peter S. and Bruce G. S. Hardie (2007b), "Fitting the sBG Model to Multi-Cohort Data." <http://brucehardie.com/notes/017/>