Appendix to:

Fader, Peter S. and Bruce G.S. Hardie (1996), "Modeling Consumer Choice Among SKUs," *Journal of Marketing Research*, **33** (November), 442–452.

Testing for Independence from Irrelevant Alternatives

One commonly discussed property of the multinomial logit model is that of independence from irrelevant alternatives. Surprisingly, very few marketing applications of the MNL model have formally tested for departures from IIA. In this paper we apply the MNL model to a reasonably heterogenous set of products and we may question whether the data support the implicit IIA assumption of the model. We now formally test the soundness of this assumption.

Most IIA tests involve the comparison of a model estimated using a full set of choice alternatives with a model estimated using a restricted set of choice alternatives. The logic behind the (first) Hausman and McFadden (1984) test is that if the IIA property is valid, the parameters of the restricted set model should be "approximately the same" as those of the full choice set model. A practical implementation issue associated with this test is that it requires the analyst to take the inverse of the difference between two closely related covariance matrices, which may be nearly singular. In many cases, therefore, the test does not work.

We have therefore chosen to use Small and Hsiao's (1985) exact test for the IIA assumption. The basic idea of this test is that if the IIA assumption holds, the (maximized) log-likelihood for the restricted choice set will not be too different from the log-likelihood computed over the restricted choice set using parameters obtained from the full choice set. [See Small and Hsiao (1985, pp. 622–3) for specific details of the test.] The resulting test statistic is asymptotically chi-square distributed with degrees of freedom equal to the number of parameters.

Published applications of IIA tests are typically based on choice sets with three or four choice alternatives, and the restricted choice set will be created by dropping one choice alternative. Some researchers will restrict themselves to a single restricted choice set, while others will test all possible restricted choice sets associated with the deletion of a single item. However, in the case of the fabric softener data set, do we wish to create the 56 restricted choice sets associated with dropping each SKU in turn, and compute the test statistic 56 times? Moreover, should we consider creating restricted choice sets with more than one item dropped? Unfortunately, the IIA testing literature provides no guidance as to how the restricted choice set(s) should be constructed.

We have chosen to create five restricted choice sets. To make the test maximally conservative, we start by creating the two restricted choice sets associated with deleting the top two best selling SKUs individually. The third restricted choice set is created by deleting the top two selling items; these two SKUs account for 14% of all choices prior to the forecast period. The last two restricted choice sets are associated with deleting three and four items. These are created by deleting the observations which divide the list of SKUs, ordered on the basis of decreasing sales, into four and five equal parts respectively (i.e., a systematic sample). In our opinion, this is the least arbitrary

approach to creating restricted choice sets with multiple items deleted and, in the spirit of our desire to be conservative, means that we avoid deleting the very low selling SKUs which would have virtually no impact on the test.

Using the single segment solution of the attribute-based model, the results of the Small and Hsiao IIA test are as follows:

Restricted Choice Set	Number of SKUs Deleted	Small & Hsiao Test Stat.
i	1	36.1^a
ii	1	22.2^b
iii	2	34.2^b
iv	3	30.7^b
v	4	36.7^a

^{a} $p > .05$; ^{b} $p > .10$

While we could create a number of other restricted choice sets, it would appear to be a pointless exercise. The values of the Small and Hsiao test statistic for the five restricted choice sets are all insignificant at the $p = .05$ level. We detect no significant departure from IIA; it appears that the IIA assumption does hold for the current model specification applied to the fabric softener dataset. Because the assumption holds for the single segment solution, we can expect IIA to be even less of a problem in the multi-segment models, which apply the MNL model to groups of relatively homogeneous consumers (Ben-Akiva and Lerman 1985; Hausman and Wise 1978).

Finally, we should remember that IIA is a property of individual choices and market shares in homogeneous populations; it does not apply to market shares in heterogeneous populations (Ben-Akiva and Lerman 1985). The fact that our model assumes IIA for any single choice occasion does not mean that our share predictions for new SKUs will suffer.

References

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